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On the structure of expansions for the BBGKY hierarchy solutions

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Abstract

We consider classical many-particle systems of identical particles and distinguishable particles. For these types of systems we construct a new representation of a solution to the initial value problem to the BBGKY hierarchy of equations, namely, in the form of an expansion over particle clusters whose evolution is governed by the cumulants (semi-invariants) of the evolution operator of the corresponding particle cluster. Such a representation of solutions enables us to describe the cluster nature of the evolution of infinite particle systems with different symmetry properties in detail. A convergence of the constructed expansions is investigated in the suitable functional spaces.

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1. Introduction

The structure of expansions for solutions of the initial value problem of the BBGKY hierarchy depends on the symmetry properties of many-particle systems which are connected with the indistinguishability property of identical particles. Classical systems of identical particles are described by observables and states which are symmetric functions with respect to permutations of their arguments (the phase space coordinates of every particle) [1, 2]. In the quantum case, we have additional symmetry properties related to the nature of identical particles (Fermi and Bose particles). Classical many-particle systems can also consist of distinguishable particles. In this case, many-particle systems are described by observables and states which are non-symmetric functions of their arguments (non-symmetrical many-particle systems).

In this paper, we construct a new representation of the solutions of the initial value problem of the BBGKY hierarchy in the form of an expansion over particle clusters whose evolution is governed by the cumulant (semi-invariant) of the evolution operator of the corresponding particle cluster for the symmetric and non-symmetric classical systems. We give existence and uniqueness results for the initial value problem in the space of sequences of integrable functions.

We note that a well-known example of initial data is the Gibbs equilibrium state [1]. The equilibrium distribution functions belong to the space of sequences of continuous functions bounded with respect to the configuration variables and momentum variables are Maxwellian distributions. As is known [1, 2], for initial data from this space there is an involved problem which involves the existence of the divergent integrals over configuration variables in each term of the solution expansion. Exactly, the stated cumulant nature of the solution expansions guarantees the compensation of the divergent integrals. A detailed analysis of this problem will be given in a separate paper.

2. Cluster expansions of evolution operators of symmetrical many-particle systems

At first we will formulate some definitions and preliminary facts about the dynamics of finite many-particle systems.

Let us introduce a set of measurable functions $f_n(x_1, \ldots, x_n)$ defined on the phase space $\mathbb{R}^{\nu n} \times \mathbb{R}^{\nu n}$, $\nu \geqslant 1$, of an n-particle system, which are invariant under permutations of the arguments $x_1, \ldots, x_n$. The dynamics of a classical system of a finite number n of particles is described by the following evolution operator:

$$\begin{aligned}(S_n(t) f_n)(x_1, \ldots, x_n) &\equiv S_n(t, x_1, \ldots, x_n) f_n(x_1, \ldots, x_n)\\ &= f_n(X_1(t, x_1, \ldots, x_n), \ldots, X_n(t, x_1, \ldots, x_n)), \end{aligned}\qquad (1)$$

where $X_i(t, x_1, \ldots, x_n), i = 1, \ldots, n,$ is a solution of the initial value problem for the Hamilton equations of a system of n particles with initial data $X_i(0, x_1, \ldots, x_n) = x_i \equiv (q_i, p_i) \in \mathbb{R}^\nu \times \mathbb{R}^\nu$, $\nu \geqslant 1, i = 1, \ldots, n$, ($S_n(0) = I$ is the identity operator). In what follows we will assume that the interaction potential Φ satisfies the necessary conditions which guarantee the existence of global in time solutions of the Hamilton equations. Examples of such conditions are given in [1, 2].

Operator (1) is defined, e.g., in the space of integrable functions $f_n \in L^1(\mathbb{R}^{\nu n} \times \mathbb{R}^{\nu n}) \equiv L^1_n$ [1] and, in particular, this operator is a strongly continuous one-parametric group of isometric operators in the space L^1_n, i.e., $\|S_n(t)\|_{L^1_n} = 1$.

On the subspace $f_n \in L^1_{n,0} \subset L^1_n$ of the continuously differentiable functions with compact supports the infinitesimal generator $\mathcal{L}_n$ of the evolution operator (1) is given by the Poisson bracket

$$\frac{\mathrm{d}}{\mathrm{d}t} S_n(t) f_n|_{t=0} = \mathcal{L}_n f_n \equiv \left(\mathcal{L}^0_n + \mathcal{L}^{\mathrm{int}}_n\right) f_n, \qquad (2)$$

where

$$\mathcal{L}^0_n = \sum_{i=1}^n \mathcal{L}^0(x_i) = \sum_{i=0}^n \left\langle p_i, \frac{\partial}{\partial q_i}\right\rangle,$$

$$\mathcal{L}^{\mathrm{int}}_n = \sum_{i<j=1}^n \mathcal{L}^{\mathrm{int}}(x_i, x_j) = \sum_{i<j=1}^n \left\langle \frac{\partial}{\partial q_i}\Phi(q_i - q_j), \left(\frac{\partial}{\partial p_j} - \frac{\partial}{\partial p_i}\right)\right\rangle$$

and $\langle \cdot, \cdot \rangle$ is a scalar product.

We consider the initial value problem of the BBGKY hierarchy of equations for a classical system of identical particles [1, 2]. If $F(0) = (1, F_1(0, x_1), \ldots, F_s(0, x_1, \ldots, x_s), \ldots)$ is a sequence of initial s-particle distribution functions $F_s(0, x_1, \ldots, x_s)$ symmetric in

$x_i \equiv (q_i, p_i) \in \mathbb{R}^\nu \times \mathbb{R}^\nu$, $\nu \geqslant 1$, then a solution $F(t) = (1, F_1(t, x_1), \ldots, F_s(t, x_1, \ldots, x_s), \ldots)$ of the Cauchy problem for the BBGKY hierarchy is represented as the expansion

$$F_s(t, x_1, \ldots, x_s) = \sum_{n=0}^{\infty} \frac{1}{n!} \int_{(\mathbb{R}^\nu\times\mathbb{R}^\nu)^n} \mathrm{d}x_{s+1} \cdots \mathrm{d}x_{s+n} \\ \times \mathfrak{A}_{(n)}(t, x_1, \ldots, x_s, x_{s+1}, \ldots, x_{s+n}) F_{s+n}(0, x_1, \ldots, x_{s+n}), \qquad s \geqslant 1, \qquad (3)$$

where the evolution operators $\mathfrak{A}_{(n)}(t)$ are defined as follows. Let $(x_1, \ldots, x_s) \equiv Y$, $(Y, x_{s+1}, \ldots, x_{s+n}) \equiv X$, i.e., $(x_{s+1}, \ldots, x_{s+n}) = X\backslash Y$, and let $|X| = |Y| + |X\backslash Y| = s + n$ denote the number of elements of the set X. Then we have

$$\mathfrak{A}_{(|X\backslash Y|)}(t, Y, X\backslash Y) = \sum_{P:\{Y, X\backslash Y\} = \bigcup_i X_i} (-1)^{|P|-1}(|P|-1)! \prod_{X_i \subset P} S_{|X_i|}(-t, X_i), \qquad |X\backslash Y| \geqslant 0 \qquad (4)$$

where $\sum_P$ is the sum over all possible decompositions of the set $\{Y, X\backslash Y\}$ into $|P|$ nonempty mutually disjoint subsets $X_i \subset \{Y, X\backslash Y\}$, $X_i \cap X_j = \varnothing$, and the set Y completely belongs to one of the subsets X_i.

The simplest examples of evolution operators $\mathfrak{A}_{(n)}(t)$ (4) have the form

$$\mathfrak{A}_{(0)}(t, Y) = S_s(-t, Y),$$
$$\mathfrak{A}_{(1)}(t, Y, x_{s+1}) = S_{s+1}(-t, Y, x_{s+1}) - S_s(-t, Y) S_1(-t, x_{s+1}),$$
$$\mathfrak{A}_{(2)}(t, Y, x_{s+1}, x_{s+2}) = S_{s+2}(-t, Y, x_{s+1}, x_{s+2}) - S_{s+1}(-t, Y, x_{s+1}) S_1(-t, x_{s+2}) \\ - S_{s+1}(-t, Y, x_{s+2}) S_1(-t, x_{s+1}) - S_s(-t, Y) S_2(-t, x_{s+1}, x_{s+2}) \\ + 2! S_s(-t, Y) S_1(-t, x_{s+1}) S_1(-t, x_{s+2}).$$

Evolution operators (4) are solutions of the following recursion relations:

$$S_{|X|}(-t, Y, X\backslash Y) = \sum_{P:\{Y, X\backslash Y\} = \bigcup_i X_i} \prod_{X_i \subset P} \mathfrak{A}_{(|X_i|-1)}(t, X_i), \qquad |X\backslash Y| \geqslant 0, \qquad (5)$$

where $\sum_P$ is the sum as above in formula (4). For example,

$$S_{|Y|}(-t, Y) = \mathfrak{A}_{(0)}(t, Y),$$
$$S_{|Y|+1}(-t, Y, x_{s+1}) = \mathfrak{A}_{(1)}(t, Y, x_{s+1}) + \mathfrak{A}_{(0)}(t, Y)\mathfrak{A}_{(0)}(t, x_{s+1}),$$
$$S_{|Y|+2}(-t, Y, x_{s+1}, x_{s+2}) = \mathfrak{A}_{(2)}(t, Y, x_{s+1}, x_{s+2}) + \mathfrak{A}_{(1)}(t, Y, x_{s+1})\mathfrak{A}_{(0)}(t, x_{s+2}) \\ + \mathfrak{A}_{(1)}(t, Y, x_{s+2})\mathfrak{A}_{(0)}(t, x_{s+1}) + \mathfrak{A}_{(0)}(t, Y)\mathfrak{A}_{(1)}(t, x_{s+1}, x_{s+2}) \\ + \mathfrak{A}_{(0)}(t, Y)\mathfrak{A}_{(0)}(t, x_{s+1})\mathfrak{A}_{(0)}(t, x_{s+2}).$$

Really, in the general case, the following lemma is true.

Lemma 1. *A solution of the recurrence relations (5) is determined by the relation*

$$\mathfrak{A}_{(|X\backslash Y|)}(t, Y, X\backslash Y) = \sum_{P:\{Y, X\backslash Y\} = \bigcup_i X_i} (-1)^{|P|-1}(|P|-1)! \prod_{X_i \subset P} S_{|X_i|}(-t, X_i), \\ n = |X\backslash Y| \geqslant 0, \qquad (6)$$

where $\sum_P$ is the sum over all possible decompositions of the set $\{Y, X\backslash Y\}$ into $|P|$ nonempty mutually disjoint subsets $X_i \subset \{Y, X\backslash Y\}$, $X_i \cap X_j = \varnothing$, and the set Y completely belongs to one of the subsets X_i.

Proof. Consider the set of sequences $\Psi = (\Psi_0, \Psi_1(x_1), \ldots, \Psi_n(x_1, \ldots, x_n), \ldots)$ of operators Ψ_n of type (1) (Ψ_0 is an operator that multiplies a function by an arbitrary number). In this set, we introduce the tensor $*$-product

$$(\Psi_1 * \Psi_2)_{|X|}(X) = \sum_{Y\subset X} (\Psi_1)_{|Y|}(Y)(\Psi_2)_{|X\backslash Y|}(X\backslash Y),$$

where $\sum_{Y\subset X}$ is the sum over all subsets Y of the set $X \equiv (x_1, \ldots, x_n)$. A similar product is used in investigating equilibrium correlation functions by the algebraic method [1]. We also introduce the following notation: $(\mathfrak{A}(t))_{1+n}(Y, x_{s+1}, \ldots, x_{s+n}) = \mathfrak{A}_{(n)}(t, Y, X\backslash Y) \equiv \mathfrak{A}_{|Y|+n}(t, Y, X\backslash Y)$.

By the definition of the $*$-product for the sequence $\mathfrak{A}(t) = (0, (\mathfrak{A}(t))_1(Y), (\mathfrak{A}(t))_2 (Y, x_{s+1}), \ldots)$, the following equality is true:

$$\sum_{P:\{Y,X\backslash Y\}=\bigcup_i X_i} \prod_{X_i\subset P} \mathfrak{A}_{(|X_i|-1)}(t, X_i) = (\mathbb{E}\mathrm{xp}_*\mathfrak{A}(t))_{1+n}(Y, X\backslash Y), \qquad n = |X\backslash Y| \geqslant 0,$$

where $\mathbb{E}\mathrm{xp}_*$ is defined as the $*$-exponential mapping, i.e.,

$$\mathbb{E}\mathrm{xp}_*\Psi = \mathbf{1} + \sum_{n=1}^{\infty} \frac{1}{n!} \underbrace{\Psi * \cdots * \Psi}_{n},$$

$\Psi \equiv (0, \Psi_1, \ldots, \Psi_n, \ldots)$ and $\mathbf{1} \equiv (1, 0, 0, \ldots)$ is the unit sequence.

As a result, we can represent the recurrence relations (5) in the form

$$\mathbf{1} + S(-t) = \mathbb{E}\mathrm{xp}_*\mathfrak{A}(t),$$

where the elements of the sequence $S(t) = (0, (S(t))_1(Y), (S(t))_2(Y, x_{s+1}), \ldots)$ are the evolution operators $(S(t))_{1+n}(Y, x_{s+1}, \ldots, x_{s+n}) \equiv S_{|Y|+n}(t, Y, x_{s+1}, \ldots, x_{s+n})$.

Similarly, defining a mapping $\mathbb{L}\mathrm{n}_*$ on the sequences $\Psi \equiv (0, \Psi_1, \ldots, \Psi_n, \ldots)$ as the mapping inverse to $\mathbb{E}\mathrm{xp}_*$, i.e.,

$$\mathbb{L}\mathrm{n}_*(\mathbf{1} + S(-t)) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \underbrace{\Psi * \cdots * \Psi}_{n},$$

we obtain

$$\sum_{P:\{Y,X\backslash Y\}=\bigcup_i X_i} (-1)^{|P|-1}(|P|-1)! \prod_{X_i\subset P} S_{|X_i|}(-t, X_i)$$
$$= \mathbb{L}\mathrm{n}_*(\mathbf{1} + S(-t))_{1+n}(Y, X\backslash Y), \qquad n = |X\backslash Y| \geqslant 0.$$

As a result, relation (6) can be rewritten in the form

$$\mathfrak{A}(t) = \mathbb{L}\mathrm{n}_*(\mathbf{1} + S(-t)), \tag{7}$$

and, therefore, expression (6) is a solution of relation (5). □

We note that the recursion relations given in (6) are typical cluster expansions [3] for the evolution operator $S_{|X|}(-t, Y, X\backslash Y)$ defined by (1). Thus, operators $\mathfrak{A}_{(|X\backslash Y|)}(t, Y, X\backslash Y)$ (4) have the meaning of the cumulants (semi-invariants) of the operator $S_{|X|}(-t, Y, X\backslash Y)$ describing the evolution of a system of a finite number $|X|$ of particles, i.e., they describe what noninteracting clusters of particles may form a system of the corresponding number of particles in the process of evolution, provided that a cluster of $|Y|$ particles evolves as a single cluster.

We remark that the connections between different representations of the BBGKY hierarchy solutions are considered in [1, 3]. For the first time several first terms of the expansions (3) and (4) for the one-particle distribution function were determined in [4, 5].

3. Cumulant representation of the BBGKY hierarchy solutions for symmetrical many-particle systems

We will consider the problem of the convergence of the expansions (3) and (4) in the space of sequences of integrable functions and prove the existence solution theorem for the initial data from this space.

Let $L^1_\alpha = \bigoplus_{n=0}^\infty \alpha^n L^1_n$ be the Banach space of sequences $f = (f_0, f_1(x_1), \ldots, f_n(x_1, \ldots, x_n), \ldots)$ of symmetric integrable functions $f_n(x_1, \ldots, x_n)$ defined on the phase space $\mathbb{R}^{\nu n} \times \mathbb{R}^{\nu n}$ with the norm

$$\|f\|_{L^1_\alpha} = \sum_{n=0}^\infty \alpha^n \|f_n\|_{L^1_n} = \sum_{n=0}^\infty \alpha^n \int_{(\mathbb{R}^\nu \times \mathbb{R}^\nu)^n} \mathrm{d}x_1 \cdots \mathrm{d}x_n |f_n(x_1, \ldots, x_n)|,$$

where $\alpha > 1$ is a positive integer; $L^1_{\alpha,0} \subset L^1_\alpha$ is the subspace of finite sequences of continuously differentiable functions with compact supports.

Since, on the sequences of integrable functions $f \in L^1_\alpha$ the annihilation operator [1, 2]

$$(\mathfrak{a} f)_n(x_1, \ldots, x_n) = \int_{\mathbb{R}^\nu \times \mathbb{R}^\nu} \mathrm{d}x_{n+1} f_{n+1}(x_1, \ldots, x_n, x_{n+1}) \tag{8}$$

is defined, in view of (7) and (8) expressions (3) and (4) take the following form in the space L^1_α

$$F(t) = \mathrm{e}^{\mathfrak{a}} \mathfrak{A}(t) F(0) = \mathrm{e}^{\mathfrak{a}} \mathbb{L}\mathrm{n}_*(\mathbf{1} + S(-t)) F(0). \tag{9}$$

For the cumulants $\mathfrak{A}_{(n)}(t)$ the following lemma is true in the space L^1_{s+n}.

Lemma 2. *If $F(0) \in L^1_{s+n}$, then the following estimate is valid:*

$$\|\mathfrak{A}_{(n)}(t) F_{s+n}(0)\|_{L^1_{s+n}} \leqslant n!\, \mathrm{e}^{n+2} \|F_{s+n}(0)\|_{L^1_{s+n}}. \tag{10}$$

Proof. According to the Liouville theorem [1, 2], we get

$$\|\mathfrak{A}_{(n)}(t) F_{s+n}(0)\|_{L^1_{s+n}} = \int \mathrm{d}x_1 \cdots \mathrm{d}x_{s+n} \Big| \sum_{P:\{Y, X\backslash Y\} = \bigcup_i X_i} (-1)^{|P|-1} (|P|-1)!$$
$$\times \prod_{X_i \subset P} S_{|X_i|}(-t, X_i) F_{s+n}(0, Y, X\backslash Y) \Big|$$
$$\leqslant \sum_{P:\{Y, X\backslash Y\} = \bigcup_i X_i} (|P|-1)! \|F_{s+n}(0)\|_{L^1_{s+n}} = \sum_{k=1}^{n+1} s(n+1, k)(k-1)! \|F_{s+n}(0)\|_{L^1_{s+n}},$$

where $s(n+1, k)$ are Stirling numbers of the second kind. Using the representation

$$s(n+1, k) = \frac{1}{k!} \sum_{\substack{r_1, \ldots, r_k \geqslant 1 \\ r_1 + \cdots + r_k = n+1}} \frac{(n+1)!}{r_1! \cdots r_k!},$$

for the numbers $s(n+1, k)$, we get

$$\sum_{k=1}^{n+1} s(n+1, k)(k-1)! = \sum_{k=1}^{n+1} \frac{1}{k} \sum_{\substack{r_1, \ldots, r_k \geqslant 1 \\ r_1 + \cdots + r_k = n+1}} \frac{(n+1)!}{r_1! \cdots r_k!} \leqslant \sum_{k=1}^{n+1} k^n$$
$$\leqslant n! \sum_{k=1}^{n+1} \mathrm{e}^k \leqslant n!\, \mathrm{e}^{n+2},$$

which yields estimate (10). □

By virtue of inequality (10), the functions defined by (3) (or (9)) satisfy the following estimate for $\alpha > \mathrm{e}$:

$$\|F(t)\|_{L^1_\alpha} \leqslant c_\alpha \|F(0)\|_{L^1_\alpha}, \tag{11}$$

where $c_\alpha = \mathrm{e}^2\left(1 - \frac{\mathrm{e}}{\alpha}\right)^{-1}$ is a constant.

Note that the parameter α can be interpreted as a quantity inverse to the density of the system, $1/v$ (the average number of particles in a unit volume). Indeed, renormalizing the functions $F_s(0) = \tilde{F}_s(0)/v^s$, we obtain expansion (3) in the parameter $1/v$. In this case, under the condition

$$\frac{1}{v} < \mathrm{e}^{-1},$$

the integrable functions $\tilde{F}_s(0) \in L^1_s$ satisfy the following inequality:

$$\|\tilde{F}(t)\|_{L^1_\alpha} \leqslant c\left(\frac{1}{v}\right)\|\tilde{F}(0)\|_{L^1_\alpha},$$

where $c\left(\frac{1}{v}\right) = \mathrm{e}^2\left(1 - \frac{\mathrm{e}}{v}\right)^{-1}$ is a constant.

Thus, according to estimate (11) the following existence theorem is true.

Theorem 1. *If $F(0) \in L^1_\alpha$ is a sequence of nonnegative functions, then for $\alpha > \mathrm{e}$, and $t \in \mathbb{R}^1$, there exists a unique solution to the initial value problem for the BBGKY hierarchy, namely, the sequence $F(t) \in L^1_\alpha$ of nonnegative functions $F_s(t)$ defined by*

$$F_{|Y|}(t, Y) = \sum_{n=0}^{\infty} \frac{1}{n!} \int \mathrm{d}(X\backslash Y) \sum_{P:\{Y, X\backslash Y\} \bigcup_i X_i} (-1)^{|P|-1}(|P|-1)!$$
$$\times \prod_{X_i \subset P} S_{|X_i|}(-t, X_i) F_{|X|}(0, Y, X\backslash Y), \tag{12}$$

where we have the same notation as for (4). This solution is a strong solution for $F(0) \in L^1_{\alpha,0}$ and a weak one for arbitrary initial data.

Proof. Let us show that expansion (3) defined in L^1_α is a strong solution of the Cauchy problem to the BBGKY hierarchy. To do this we first differentiate the functions $F_{|Y|}(t, Y)$ with respect to time using point-by-point convergence. Let $F(0) \in L^1_{\alpha,0}$, then, according definitions (2) and (4), for each fixed point Y we obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathfrak{A}_{(n)}(t, Y, X\backslash Y) F_{|X|}(0, X) = \sum_{P:\{Y, X\backslash Y\} = \bigcup_i X_i} (-1)^{|P|-1}(|P|-1)!$$
$$\times \sum_{X_i \subset P} (-\mathcal{L}_{|X_i|}(X_i)) \prod_{X_i \subset P} S_{|X_i|}(-t, X_i) F_{|X|}(0, X).$$

In view of the equality

$$\sum_{X_i \subset P} \mathcal{L}_{|X_i|}(X_i) = \mathcal{L}_{|Y \bigcup X\backslash Y|} - \sum_{X_{j_1}, X_{j_2} \subset P} \mathcal{L}^{\mathrm{int}}_{|X_{j_1} \bigcup X_{j_2}|}\left(X_{j_1}; X_{j_2}\right),$$

where

$$\mathcal{L}^{\mathrm{int}}_{|X_{j_1} \bigcup X_{j_2}|}\left(X_{j_1}; X_{j_2}\right) = \sum_{i_1=1}^{|X_{j_1}|} \sum_{i_1=1}^{|X_{j_2}|} \mathcal{L}^{\mathrm{int}}\left(x_{i_1}, x_{i_2}\right)$$

and the validity of the following identity:

$$\int \mathrm{d}(X\backslash Y)\mathcal{L}^{\mathrm{int}}_{|X_{j_1}\bigcup X_{j_2}|}\big(X_{j_1}; X_{j_2}\big)\prod_{X_i\subset P} S_{|X_i|}(-t, X_i)F_{|X|}(0, X) = 0,$$

which is a result of an integration by parts on momentum variables of the integrant according to definition (2) of the operator $\mathcal{L}^{\mathrm{int}}_{|X_{j_1}\bigcup X_{j_2}|}$ and since $F(0) \in L^1_{\alpha,0}$ then for the integrant we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathfrak{A}_{(n)}(t, Y, X\backslash Y)F_{|X|}(0, X) = -\mathcal{L}_{|Y\bigcup X\backslash Y|}\mathfrak{A}_{(n)}(t, Y, X\backslash Y)F_{|X|}(0, X).$$

Therefore, for solution expansion (3) we derive

$$\frac{\mathrm{d}}{\mathrm{d}t}F(t) = \mathrm{e}^{\mathfrak{a}}(-\mathcal{L})\mathfrak{A}(t)F(0) = \mathrm{e}^{\mathfrak{a}}(-\mathcal{L})\,\mathrm{e}^{-\mathfrak{a}}\,\mathrm{e}^{\mathfrak{a}}\mathfrak{A}(t)F(0) = \mathrm{e}^{\mathfrak{a}}(-\mathcal{L})\,\mathrm{e}^{-\mathfrak{a}}F(t),$$

i.e., the sequence $F(t)$ satisfies the following hierarchy of equations (BBGKY hierarchy):

$$\frac{\mathrm{d}}{\mathrm{d}t}F(t) = \mathrm{e}^{\mathfrak{a}}(-\mathcal{L})\,\mathrm{e}^{-\mathfrak{a}}F(t).$$

If $f \in L^1_{\alpha,0}$, for the pair interaction potential Φ, the following identity is valid [2] $[[\mathcal{L}, \mathfrak{a}], \mathfrak{a}]f = 0$, where $[\cdot\,,\ \cdot]$ is a commutator of operators. Then for the generator $\mathrm{e}^{\mathfrak{a}}(-\mathcal{L})\,\mathrm{e}^{-\mathfrak{a}}$ of the BBGKY hierarchy of equations we have

$$\mathrm{e}^{\mathfrak{a}}(-\mathcal{L})\,\mathrm{e}^{-\mathfrak{a}} = -\mathcal{L} + [\mathcal{L}, \mathfrak{a}],$$

where

$$([\mathcal{L}, \mathfrak{a}]f)_n = \int \mathrm{d}x_{n+1}\sum_{i=1}^{n}\left\langle \frac{\partial}{\partial q_i}\Phi(q_i - q_{n+1}), \frac{\partial}{\partial p_i}\right\rangle f_{n+1},$$

i.e., for the smooth interaction potential we derive the accepted form of the BBGKY hierarchy. According to the well-known theorem of functional analysis [2] function $F(t)$ is differentiable in the norm of the space L^1_α for $F(0) \in L^1_{\alpha,0}$ and thus, for these initial data the corresponding Cauchy problem has a unique strong solution defined by expansions (3) and (4).

A statement about the nonnegativity of solution (3) follows from the fact that the operator $\mathfrak{A}_{(n)}(t, Y, X\backslash Y)$ is a solution of recurrence relations (5) and from definition (1) of the evolution operator $S_{|X|}(-t, Y, X\backslash Y)$. □

4. Cluster expansions of evolution operators of non-symmetrical many-particle systems

We consider a one-dimensional system of identical particles interacting with their nearest neighbours via the hard-core pair potential Φ. For the configurations ($q_i \in \mathbb{R}^1$ is the position of the centre of particle i) of such a system the following inequalities must be satisfied: $\sigma + q_i \leqslant q_{i+1}$, where σ is the length of a particle, and the natural way to number the particles is to number by means of the integers from the set $\mathbb{Z}\backslash\{0\}$. The set $W_{n_1+n_2} \equiv \{(q_{-n_2}, \ldots, q_{-1}, q_1, \ldots, q_{n_1}) \in \mathbb{R}^{n_1+n_2} \mid \sigma + q_i > q_{i+1}$ for at least one pair $(i, i+1) \in ((-n_2, -n_2+1), \ldots, (-1, 1), \ldots, (n_1-1, n_1))\}$ is the set of forbidden configurations. The Hamiltonian of the $n = n_1 + n_2$ particle system

$$H_n = \sum_{i\in(-n_2,\ldots,-1,1,\ldots,n_1)}\frac{p_i^2}{2} + \sum_{(i,i+1)\in((-n_2,-n_2+1),\ldots,(n_1-1,n_1))}\Phi(q_i - q_{i+1})$$

is a function non-symmetrical [6, 7] with respect to permutations of the arguments $x_i \equiv (q_i, p_i) \in \mathbb{R}^1\times\mathbb{R}^1$.

If $F(0) = \{F_s(0, x_{-s_2}, \ldots, x_{s_1})\}_{s=s_1+s_2\geqslant 0}$ is a sequence of initial s-particle distribution functions $F_s(0, x_{-s_2}, \ldots, x_{-1}, x_1, \ldots, x_{s_1})$, $F_0 = 1$, then a solution of the Cauchy problem for the BBGKY hierarchy $F(t) = \{F_s(t, x_{-s_2}, \ldots, x_{s_1})\}_{s=s_1+s_2\geqslant 0}$ is represented as the expansion

$$F_s(t, x_{-s_2}, \ldots, x_{s_1}) = \sum_{n=0}^{\infty} \sum_{\substack{n=n_1+n_2\\ n_1, n_2\geqslant 0}} \int_{(\mathbb{R}^1\times\mathbb{R}^1)^{n_1+n_2}} \mathrm{d}x_{-(n_2+s_2)} \cdots \mathrm{d}x_{-(s_2+1)} \times \mathrm{d}x_{s_1+1} \cdots \mathrm{d}x_{s_1+n_1} \big(\mathfrak{A}_{(n_2,n_1)}(t) F_{s+n}(0)\big)\big(x_{-(n_2+s_2)}, \ldots, x_{s_1+n_1}\big), \qquad (13)$$

where the evolution operator $\mathfrak{A}_{(n_2,n_1)}(t)$ is defined in the following way. Let $(x_{-s_2}, \ldots, x_{s_1}) \equiv Y$, $(x_{-(n_2+s_2)}, \ldots, x_{s_1+n_1}) \equiv X$. The sets X and Y are partially ordered sets, because $\sigma + q_i \leqslant q_{i+1}$. If the subset Y of the set X is treated as one element similar to $(x_{-(n_2+s_2)}, \ldots, x_{-(s_2+1)}, x_{s_1+1}, \ldots, x_{s_1+n_1})$, then for such a partially ordered set we use the notation X_Y. Symbol $|Y| = s = s_1 + s_2$ denotes the number of elements of the set Y and, thus, $|X_Y| = n_1 + n_2 + 1$. Then we have

$$\mathfrak{A}_{(n_2,n_1)}(t, X_Y) = \sum_{P: X_Y=\bigcup_i X_i} (-1)^{|P|-1} \prod_{X_i\subset P} S_{|X_i|}(-t, X_i), \qquad (14)$$

where $n_1 + n_2 = n \geqslant 0$, $\sum_P$ is the sum over all ordered decompositions of the partially ordered set X_Y into $|P|$ nonempty partially ordered subsets $X_i \subset X_Y$, which are mutually disjoint $X_i \bigcap X_j = \varnothing$, and the set Y completely belongs to one of the subsets X_i. As above, in formula (14) the evolution operator $S_{|X|}(-t, X)$ describes the dynamics of a system of a finite number $n = n_1 + n_2$ of particles [2, 6]. The simplest examples of evolution operators (14) have the form

$$\mathfrak{A}_{(0,0)}(t, Y) = S_s(-t, Y),$$
$$\mathfrak{A}_{(0,1)}(t, Y, x_{s_1+1}) = S_{s+1}(-t, Y, x_{s_1+1}) - S_s(-t, Y) S_1(-t, x_{s_1+1}),$$
$$\mathfrak{A}_{(1,0)}(t, x_{-(s_2+1)}, Y) = S_{s+1}(-t, x_{-(s_2+1)}, Y) - S_s(-t, Y) S_1(-t, x_{-(s_2+1)}),$$
$$\mathfrak{A}_{(0,2)}(t, Y, x_{s_1+1}, x_{s_1+2}) = S_{s+2}(-t, Y, x_{s_1+1}, x_{s_2+2}) - S_{s+1}(-t, Y, x_{s_1+1}) S_1(-t, x_{s_1+2}) - S_s(-t, Y) S_2(-t, x_{s_1+1}, x_{s_1+2}) + S_s(-t, Y) S_1(-t, x_{s_1+1}) S_1(-t, x_{s_1+2}),$$
$$\mathfrak{A}_{(1,1)}(t, x_{-(s_2+1)}, Y, x_{s_1+1}) = S_{s+2}(-t, x_{-(s_2+1)}, Y, x_{s_1+1}) - S_1(-t, x_{-(s_2+1)}) S_{s+1}(-t, Y, x_{s_1+1}) - S_{s+1}(-t, x_{-(s_2+1)}, Y) S_1(-t, x_{s_1+1}) + S_1(-t, x_{-(s_2+1)}) S_s(-t, Y) S_1(-t, x_{s_1+1}).$$

The evolution operators (14) are solutions of the following recursion relations:

$$S_{|X|}(-t, X) = \sum_{P: X_Y=\bigcup_i X_i} \prod_{X_i\subset P} \mathfrak{A}_{(i_2,i_1)}(t, X_i), \qquad |X_Y| - 1 \geqslant 0, \qquad (15)$$

where $i_1 + i_2 = i = |X_i| - 1 \geqslant 0$, and $\sum_P$ is the sum given above in formula (14). For example,

$$S_{|Y|}(-t, Y) = \mathfrak{A}_{(0,0)}(t, Y),$$
$$S_{|Y|+1}(-t, Y, x_{s_1+1}) = \mathfrak{A}_{(0,1)}(t, Y, x_{s_1+1}) + \mathfrak{A}_{(0,0)}(t, Y)\mathfrak{A}_{(0,0)}(t, x_{s_1+1}),$$
$$S_{|Y|+1}(-t, x_{-(s_2+1)}, Y) = \mathfrak{A}_{(1,0)}(t, x_{-(s_2+1)}, Y) + \mathfrak{A}_{(0,0)}(t, Y)\mathfrak{A}_{(0,0)}(t, x_{-(s_2+1)}),$$
$$S_{|Y|+2}(-t, Y, x_{s_1+1}, x_{s_2+2}) = \mathfrak{A}_{(0,2)}(t, Y, x_{s_1+1}, x_{s_1+2}) + \mathfrak{A}_{(0,1)}(t, Y, x_{s_1+1})\mathfrak{A}_{(0,0)}(t, x_{s_1+2}) + \mathfrak{A}_{(0,0)}(t, Y)\mathfrak{A}_{(0,1)}(t, x_{s_1+1}, x_{s_1+2}) + \mathfrak{A}_{(0,0)}(t, Y)\mathfrak{A}_{(0,0)}(t, x_{s_1+1})\mathfrak{A}_{(0,0)}(t, x_{s_1+2}),$$

$$S_{|Y|+2}\big(-t, x_{-(s_2+1)}, Y, x_{s_1+1}\big) = \mathfrak{A}_{(1,1)}\big(t, x_{-(s_2+1)}, Y, x_{s_1+1}\big)$$
$$+ \mathfrak{A}_{(0,0)}\big(t, x_{-(s_2+1)}\big)\mathfrak{A}_{(0,1)}\big(t, Y, x_{s_1+1}\big) + \mathfrak{A}_{(1,0)}\big(t, x_{-(s_2+1)}, Y\big)\mathfrak{A}_{(0,0)}\big(t, x_{s_1+1}\big)$$
$$+ \mathfrak{A}_{(0,0)}\big(t, x_{-(s_2+1)}\big)\mathfrak{A}_{(0,0)}(t, Y)\mathfrak{A}_{(0,0)}\big(t, x_{s_1+1}\big).$$

The recursion relations (15) are cluster expansions for the evolution operator of non-symmetrical particle systems. We note that the structure of cluster expansion (15) is essentially different from the structure of corresponding expansion (5) for the symmetrical systems.

As above, the structure of the cluster expansions (15) can be represented in a more explicit and compact form. In the set of double sequences $\Psi = \{\Psi_{n_1+n_2}(x_{-n_2}, \ldots, x_{n_1})\}_{n_1+n_1=n\geqslant 0}$ of operators $\Psi_{n_1+n_2}$ we introduce the following tensor $\star$-product:

$$(\Psi_1 \star \Psi_2)_{|X|}(X) = \sum_{Y\subset X} (\Psi_1)_{|Y|}(Y)(\Psi_2)_{|X\backslash Y|}(X\backslash Y),$$

where $\sum_{Y\subset X}$ is the sum over all partially ordered subsets Y of the partially ordered set $X \equiv \big(x_{-n_2}, \ldots, x_{n_1}\big)$.

Then expression (14) for the cumulants of non-symmetrical systems can be rewritten in the form

$$\mathfrak{A}(t) = \mathbf{1} - (\mathbf{1} + S(-t))^{-\mathbf{1}_\star}, \tag{16}$$

where $\mathfrak{A}(t) = (0, (\mathfrak{A}(t))_1(Y), \ldots, (\mathfrak{A}(t))_{1+n_1+n_2}(X_Y), \ldots)$ and $(\mathfrak{A}(t))_{1+n_1+n_2}\big(x_{-(n_2+s_2)}, \ldots, x_{-(s_2+1)}, Y, x_{s_1+1}, \ldots, x_{s_1+n_1}\big) \equiv (\mathfrak{A}(t))_{1+n}(X_Y) = \mathfrak{A}_{(n_2,n_1)}(t, X_Y)$. A mapping $\mathbf{1} - (\mathbf{1} + \cdot)^{-\mathbf{1}_\star}$ is defined then by the formula

$$\mathbf{1} - (\mathbf{1} + \Psi)^{-\mathbf{1}_\star} = \sum_{n=1}^{\infty} (-1)^{n-1} \underbrace{\Psi \star \cdots \star \Psi}_{n}$$

$\big(\Psi \equiv (0, \Psi_1, \ldots, \Psi_{1+n_1+n_2}, \ldots\big)$ and $\mathbf{1} = (1, 0, 0, \ldots)$ is the unit sequence).

The cluster expansions (15) for the evolution operator of non-symmetrical particle systems have the form

$$\mathbf{1} + S(-t) = (\mathbf{1} - \mathfrak{A}(t))^{-\mathbf{1}_\star},$$

where $(\mathbf{1} - \cdot)^{-\mathbf{1}_\star}$ is defined as the $\star$-resolvent:

$$(\mathbf{1} - \Psi)^{-\mathbf{1}_\star} = \mathbf{1} + \sum_{n=1}^{\infty} \underbrace{\Psi \star \cdots \star \Psi}_{n}.$$

5. Cumulant representation of the BBGKY hierarchy solutions for non-symmetrical many-particle systems

We consider the problem of the convergence of expansions (13) and (14) in the space of sequences of integrable functions. Let

$$L^1_\alpha = \bigoplus_{n=0}^{\infty} \bigoplus_{\substack{n=n_1+n_2\\ n_1, n_2\geqslant 0}} \alpha^{n_1+n_2} L^1_{n_1+n_2}$$

be the Banach space of double sequences $f = \big\{f_n\big(x_{-n_2}, \ldots, x_{n_1}\big)\big\}_{n=n_1+n_2\geqslant 0}$ of integrable functions $f_n\big(x_{-n_2}, \ldots, x_{n_1}\big)$ defined on the phase space $\mathbb{R}^n \times (\mathbb{R}^n \setminus W_n)$ [2, 7] with the norm

$$\|f\|_{L^1_\alpha} = \sum_{n=0}^{\infty} \sum_{\substack{n=n_1+n_2\\ n_1, n_2\geqslant 0}} \alpha^{n_1+n_2} \int_{(\mathbb{R}^1\times\mathbb{R}^1)^{n_1+n_2}} \mathrm{d}x_{-n_2} \cdots \mathrm{d}x_{n_1} \big|f_{n_1+n_2}\big(x_{-n_2}, \ldots, x_{n_1}\big)\big|,$$

where $\alpha > 1$ is a number; $L^1_{\alpha,0} \subset L^1_\alpha$ is the subspace of finite sequences of continuously differentiable functions with compact supports.

Since, on the sequences of integrable functions $f \in L^1_\alpha$ the operators

$$(\mathfrak{a}_{(+)}f)_n\big(x_{-n_2},\ldots,x_{n_1}\big) = \int_{\mathbb{R}^1\times\mathbb{R}^1} \mathrm{d}x_{n_1+1}\, f_{n+1}\big(x_{-n_2},\ldots,x_{n_1},x_{n_1+1}\big),$$
$$(\mathfrak{a}_{(-)}f)_n\big(x_{-n_2},\ldots,x_{n_1}\big) = \int_{\mathbb{R}^1\times\mathbb{R}^1} \mathrm{d}x_{-(n_2+1)}\, f_{n+1}\big(x_{-(n_2+1)},x_{-n_2},\ldots,x_{n_1}\big)$$

are defined, in view of (16) expansion (13) takes the following form in the space L^1_α:

$$\begin{aligned} F(t) &= (\mathbf{1}-\mathfrak{a}_{(+)})^{-1}(\mathbf{1}-\mathfrak{a}_{(-)})^{-1}\mathfrak{A}(t)F(0) \\ &= (\mathbf{1}-\mathfrak{a}_{(+)})^{-1}(\mathbf{1}-\mathfrak{a}_{(-)})^{-1}(\mathbf{1}-(\mathbf{1}+S(-t))^{-\mathbf{1},\star})F(0). \end{aligned}$$

For the cumulants $\mathfrak{A}_{(n_2,n_1)}(t)$ the following lemma is true in the space L^1_{s+n}.

Lemma 3. *If $F(0) \in L^1_{s+n}$, then the following estimate is valid:*

$$\|\mathfrak{A}_{(n_2,n_1)}(t)F_{s+n}(0)\|_{L^1_{s+n}} \leqslant 2^{n_1+n_2}\|F_{s+n}(0)\|_{L^1_{s+n}}.$$

Proof. According to the Liouville theorem [2], we get

$$\begin{aligned} \|\mathfrak{A}_{(n_2,n_1)}(t)F_{s+n}(0)\|_{L^1_{s+n}} &= \int \mathrm{d}X \Big| \sum_{P:X_Y=\bigcup_i X_i} (-1)^{|P|-1} \prod_{X_i\subset P} S_{|X_i|}(-t,X_i)F_{s+n}(0,X)\Big| \\ &\leqslant \sum_{P:X_Y=\bigcup_i X_i} \|F_{s+n}(0)\|_{L^1_{s+n}} = 2^{n_1+n_2}\|F_{s+n}(0)\|_{L^1_{s+n}}, \end{aligned}$$

where the last equality follows from the fact that the number of ordered decompositions $P : X_Y = \bigcup_i X_i$ of the set X_Y consisting of n_1+n_2+1 elements equals $2^{n_1+n_2}$. □

By virtue of this inequality, the functions $F(t)$ defined by (13) and (14) satisfy the following estimate for $\alpha > 2$;

$$\|F(t)\|_{L^1_\alpha} \leqslant c^2_\alpha \|F(0)\|_{L^1_\alpha}, \tag{17}$$

where $c_\alpha = \big(1-\frac{2}{\alpha}\big)^{-1}$ is a constant.

Thus, according to (17) the following existence theorem holds.

Theorem 2. *If $F(0) \in L^1_\alpha$ is a sequence of nonnegative functions, then for $\alpha > 2$, and $t \in \mathbb{R}^1$, there exists a unique strong solution of the initial value problem for the BBGKY hierarchy for $F(0) \in L^1_{\alpha,0}$, namely, the sequence $F(t) \in L^1_\alpha$ of nonnegative functions $F_{s_1+s_2}(t)$ defined by*

$$\begin{aligned} F_{|Y|}(t,Y) = \sum_{n=0}^{\infty} \sum_{\substack{n=n_1+n_2 \\ n_1,n_2\geqslant 0}} \int_{(\mathbb{R}^1\times\mathbb{R}^1)^{n_1+n_2}} \mathrm{d}(X\backslash Y) \sum_{P:X_Y=\bigcup_i X_i} (-1)^{|P|-1} \\ \times \prod_{X_i\subset P} S_{|X_i|}(-t,X_i)F_{|X|}(0,X), \end{aligned}$$

where we have the same notation as for relations (12).

Proof. The statement of the theorem is proved similar to theorem 1. Indeed, if $F(0) \in L^1_{\alpha,0}$, then for the integrant we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathfrak{A}_{(n)}(t,Y,X_Y)F_{|X|}(0,X) = -\mathcal{L}_{|Y\bigcup X_Y|}\mathfrak{A}_{(n)}(t,Y,X_Y)F_{|X|}(0,X).$$

Therefore, for solution expansion (13) we derive

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}F(t) &= (\mathbf{1}-\mathfrak{a}_{(+)})^{-1}(\mathbf{1}-\mathfrak{a}_{(-)})^{-1}(-\mathcal{L})\mathfrak{A}(t)F(0)\\ &= (\mathbf{1}-\mathfrak{a}_{(+)})^{-1}(\mathbf{1}-\mathfrak{a}_{(-)})^{-1}(-\mathcal{L})(\mathbf{1}-\mathfrak{a}_{(+)})(\mathbf{1}-\mathfrak{a}_{(-)})\\ &\quad\times(\mathbf{1}-\mathfrak{a}_{(+)})^{-1}(\mathbf{1}-\mathfrak{a}_{(-)})^{-1}\mathfrak{A}(t)F(0)\\ &= (\mathbf{1}-\mathfrak{a}_{(+)})^{-1}(\mathbf{1}-\mathfrak{a}_{(-)})^{-1}(-\mathcal{L})(\mathbf{1}-\mathfrak{a}_{(+)})(\mathbf{1}-\mathfrak{a}_{(-)})F(t),\end{aligned}$$

i.e., the sequence $F(t)$ satisfies the following hierarchy of equations (BBGKY hierarchy):

$$\frac{\mathrm{d}}{\mathrm{d}t}F(t) = (\mathbf{1}-\mathfrak{a}_{(+)})^{-1}(\mathbf{1}-\mathfrak{a}_{(-)})^{-1}(-\mathcal{L})(\mathbf{1}-\mathfrak{a}_{(+)})(\mathbf{1}-\mathfrak{a}_{(-)})F(t).$$

If $f \in L^1_{\alpha,0}$, for the pair interaction potential Φ, the following identities are valid [2] $(\mathfrak{a}_{(\mp)})^n[\mathcal{L},\mathfrak{a}_{(\pm)}]f = 0, n \geqslant 1$, where $[\cdot,\cdot]$ is a commutator of operators. Then for the generator $(\mathbf{1}-\mathfrak{a}_{(+)})^{-1}(\mathbf{1}-\mathfrak{a}_{(-)})^{-1}(-\mathcal{L})(\mathbf{1}-\mathfrak{a}_{(+)})(\mathbf{1}-\mathfrak{a}_{(-)})$ of the BBGKY hierarchy we have

$$(\mathbf{1}-\mathfrak{a}_{(+)})^{-1}(\mathbf{1}-\mathfrak{a}_{(-)})^{-1}(-\mathcal{L})(\mathbf{1}-\mathfrak{a}_{(+)})(\mathbf{1}-\mathfrak{a}_{(-)}) = -\mathcal{L}+[\mathcal{L},\mathfrak{a}_{(+)}]+[\mathcal{L},\mathfrak{a}_{(-)}],$$

where

$$\begin{aligned}([\mathcal{L},\mathfrak{a}_{(+)}]f)_n &= \int \mathrm{d}x_{n_1+1}\left\langle \frac{\partial}{\partial q_{n_1}}\Phi\big(q_{n_1}-q_{n_1+1}\big), \frac{\partial}{\partial p_{n_1}}\right\rangle f_{n_2+n_1+1}\\ &\quad+\int_0^\infty \mathrm{d}P\,P\big(f_{n_2+n_1+1}\big(t,x_{-n_2},\ldots,x_{n_1-1};q_{n_1},p_{n_1}+P;q_{n_1}+\sigma,p_{n_1}\big)\\ &\quad-f_{n_2+n_1+1}\big(t,x_{-n_2},\ldots,x_{n_1};q_{n_1}+\sigma,p_{n_1}-P\big)\big),\\ ([\mathcal{L},\mathfrak{a}_{(-)}]f)_n &= \int \mathrm{d}x_{-(n_2+1)}\left\langle \frac{\partial}{\partial q_{-n_2}}\Phi\big(q_{-n_2}-q_{-(n_2+1)}\big), \frac{\partial}{\partial p_{-n_2}}\right\rangle f_{n_2+1+n_1}\\ &\quad+\int_0^\infty \mathrm{d}P\,P\big(f_{n_2+1+n_1}\big(t,q_{-n_2}-\sigma,p_{-n_2};q_{-n_2},p_{-n_2}-P;x_{-(n_2-1)},\ldots,x_{n_1}\big)\\ &\quad-f_{n_2+1+n_1}\big(t,q_{-n_2}-\sigma,p_{-n_2}+P;x_{-n_2},\ldots,x_{n_1}\big)\big),\end{aligned}$$

i.e., for the smooth interaction potential we derive the accepted form of the BBGKY hierarchy [7]. According to the well-known theorem of functional analysis [2] function $F(t)$ is differentiable in the norm of the space L^1_α for $F(0) \in L^1_{\alpha,0}$ and thus, for these initial data the corresponding Cauchy problem has a unique strong solution defined by expansions (13) and (14).

□

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